

21.19. Model: A string fixed at both ends forms standing waves.

Solve: A simple string sounds the fundamental frequency $f_1 = v/2L$. Initially, when the string is of length $L_A = 30$ cm, the note has the frequency $f_{1A} = v/2L_A$. For a different length, $f_{1B} = v/2L_B$. Taking the ratio of each side of these two equations gives

$$\frac{f_{1A}}{f_{1B}} = \frac{v/2L_A}{v/2L_B} = \frac{L_B}{L_A} \Rightarrow L_B = \frac{f_{1A}}{f_{1B}} L_A$$

We know that the second frequency is desired to be $f_{1B} = 523$ Hz. The string length must be

$$L_B = \frac{440 \text{ Hz}}{523 \text{ Hz}}(30 \text{ cm}) = 25.2 \text{ cm}$$

The question is not how long the string must be, but where must the violinist place his finger. The full string is 30 cm long, so the violinist must place his finger 4.8 cm from the end.

Assess: A fingering distance of 4.8 cm from the end is reasonable.